

CBSE Class 10 Mathematics

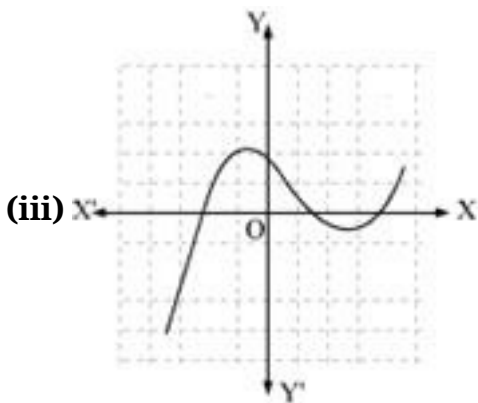
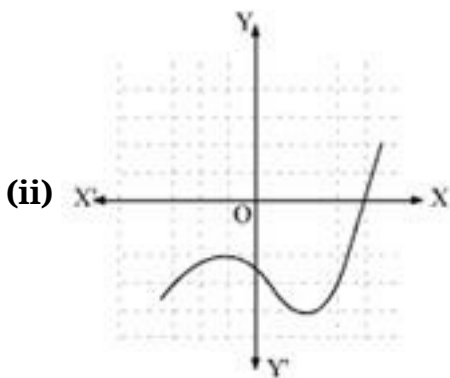
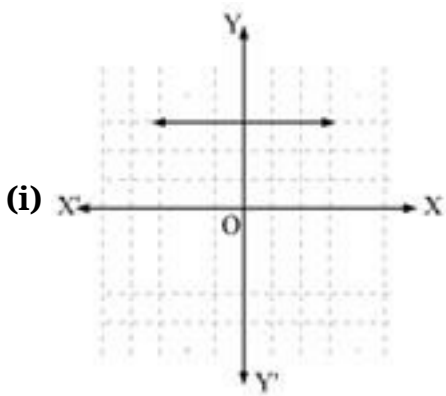
Important Questions

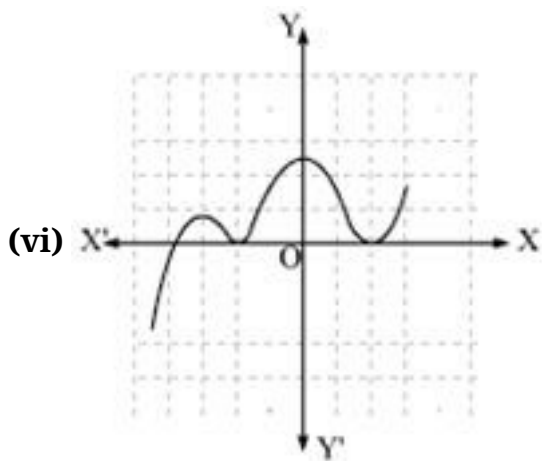
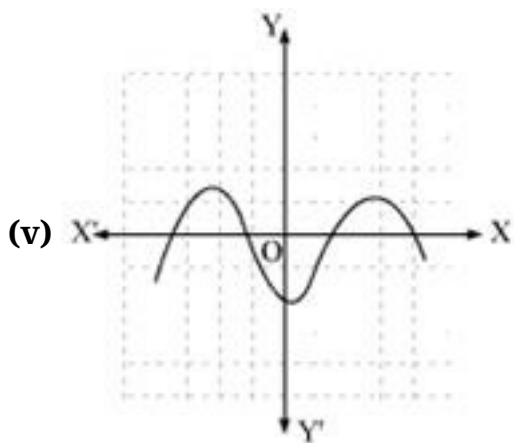
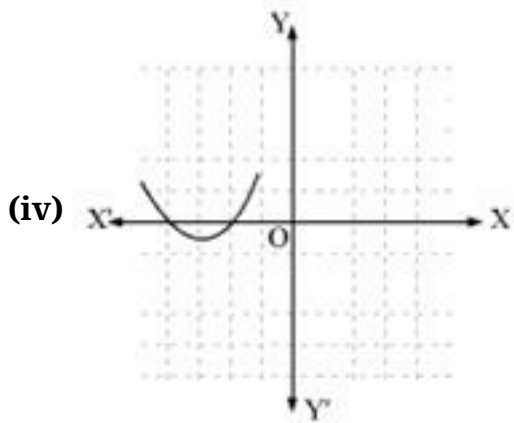
Chapter 2

Polynomials

1 Marks Questions

1. The graphs of  $y=p(x)$  are given to us, for some polynomials  $p(x)$ . Find the number of zeroes of  $p(x)$ , in each case.





**Ans. (i)** The graph does not meet the x-axis at all. Hence, it does not have any zero.

**(ii)** Graph meets x-axis 1 time. It means this polynomial has 1 zero.

**(iii)** Graph meets x-axis 3 times. Therefore, it has 3 zeroes.

**(iv)** Graph meets x-axis 2 times. Therefore, it has 2 zeroes.

(v) Graph meets x-axis 4 times. It means it has 4 zeroes.

(vi) Graph meets x-axis 3 times. It means it has 3 zeroes

**2. Which of the following is polynomial?**

(a)  $x^2 - 6\sqrt{x} + 2$

(b)  $\sqrt{x} + \frac{1}{\sqrt{x}}$

(c)  $\frac{5}{x^2 - 3x + 1}$

(d) none of these

Ans. (d) none of these

**3. Polynomial  $2x^4 + 3x^3 - 5x^2 - 5x^2 + 9x + 1$  is a**

(a) linear polynomial

(b) quadratic polynomial

(c) cubic polynomial

(d) bi-quadratic polynomial

Ans. (d) bi-quadratic polynomial

**4. If  $\alpha$  and  $\beta$  are zeros of  $x^2 + 5x + 8$ , then the value of  $(\alpha + \beta)$  is**

(a) 5

(b) -5

(c) 8

(d) -8

**Ans. (b) -5**

**5. The sum and product of the zeros of a quadratic polynomial are 2 and -15 respectively. The quadratic polynomial is**

**(a)  $x^2 - 2x + 15$**

**(b)  $x^2 - 2x - 15$**

**(c)  $x^2 + 2x - 15$**

**(d)  $x^2 + 2x + 15$**

**Ans. (b)  $x^2 - 2x - 15$**

**6. If  $p(x) = 2x^2 - 3x + 5$ ,  $3x + 5$ , then  $P(-1)$  is equal to**

**(a) 7**

**(b) 8**

**(c) 9**

**(d) 10**

**Ans. (d) 10**

**7. Zeros of  $p(x) = x^2 - 2x - 3$  are**

**(a) 3 and 1**

**(b) 3 and -1**

**(c) -3 and -1**

**(d) 1 and -3**

**Ans. (b) 3 and -1**

8. If  $\alpha$  and  $\beta$  are the zeros of  $2x^2 + 5x - 10$ , then the value of  $\alpha\beta$  is

(a)  $-\frac{5}{2}$

(b) 5

(c) -5

(d)  $\frac{2}{5}$

Ans. (c)-5

9. A quadratic polynomial, the sum and product of whose zeros are 0 and  $\sqrt{5}$  respectively is

(a)  $x^2 + \sqrt{5}$

(b)  $x^2 - \sqrt{5}$

(c)  $x^2 - 5$

(d) None of these

Ans. a)  $x^2 + \sqrt{5}$

10. Which of the following is polynomial?

(a)  $x^2 - 6\sqrt{x} + 2$

(b)  $\sqrt{x} + \frac{1}{\sqrt{x}}$

(c)  $\frac{5}{x^2 - 3x + 1}$

(d) none of these

Ans. (d) none of these

11. Polynomial  $2x^4 + 3x^3 - 5x^2 - 5x^2 + 9x + 1$  is a

(a) linear polynomial

(b) quadratic polynomial

(c) cubic polynomial

(d) bi-quadratic polynomial

Ans. (d) bi-quadratic polynomial

12. If  $\alpha$  and  $\beta$  are zeros of  $x^2 + 5x + 8$ , then the value of  $(\alpha + \beta)$  is

(a) 5

(b) -5

(c) 8

(d) -8

Ans. (b) -5

13. The sum and product of the zeros of a quadratic polynomial are 2 and -15 respectively. The quadratic polynomial is

(a)  $x^2 - 2x + 15$

(b)  $x^2 - 2x - 15$

(c)  $x^2 + 2x - 15$

(d)  $x^2 + 2x + 15$

Ans. (b)  $x^2 - 2x - 15$

CBSE Class 10 Mathematics

Important Questions

Chapter 2

Polynomials

2 Marks Questions

1. Find the quadratic polynomial where sum and product of the zeros are  $a$  and  $\frac{1}{a}$ .

Ans. Polynomial  $x^2 - 9x + \frac{1}{9}$  i.e.  $\frac{1}{9}[9x^2 - 9^2x + 1]$

2. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $f(x) = x^2 - x - 4$ , find the value of  $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$ .

Ans.  $f(x) = x^2 - x - 4$  i.e.

If  $\alpha$  and  $\beta$  are the zeroes

$$\therefore \alpha + \beta = \frac{1}{1} = 1$$

$$\alpha \cdot \beta = \frac{-4}{1} = -4$$

So,

$$\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{\alpha + \beta}{\alpha\beta} - \alpha\beta$$

$$= \frac{1}{-4} - (-4)$$

$$= -\frac{1}{4} + 4$$

$$= \frac{15}{4}$$

3. If the square of the difference of the zeros of the quadratic polynomial  $f(x) = x^2 + px + 45$  is equal to 144, find the value of  $p$ .

Ans.  $\alpha + \beta = -p$   
 $\alpha\beta = 45$

$$(\alpha - \beta)^2 = 144$$

$$\Rightarrow \alpha^2 + \beta^2 - 2\alpha\beta$$

$$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 144$$

$$\Rightarrow (-p)^2 - 4 \times 45 = 144$$

$$\Rightarrow p^2 = 144 + 180$$

$$\Rightarrow p = \pm 18$$

4. Divide  $(6x^3 - 26x - 21 + x^2)$  by  $(-7 + 3x)$ .

Ans.

$$\begin{array}{r}
 2x^2 + 5x + 3 \\
 3x - 7 \overline{) 6x^3 + x^2 - 26x - 21} \\
 \underline{-6x^3 + 14x^2} \phantom{-21} \\
 15x^2 - 26x - 21 \\
 \underline{-15x^2 + 35x} \phantom{-21} \\
 9x - 21 \\
 \underline{-9x + 21} \\
 0
 \end{array}$$

$\therefore$  quotient =  $2x^2 + 5x + 3$

5. Find the value of 'k' such that the quadratic polynomial  $x^2 - (k + 6)x + 2(2k + 1)$  has sum of the zeros is half of their product.



**Ans.** Sum of the zeros =  $\frac{1}{2}$  product of the zeros

$$\Rightarrow (k+6) = \frac{1}{2} [2(2k+1)]$$

$$\Rightarrow k+6 = 2k+1$$

$$\Rightarrow k = 5$$

**6.** If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $f(x) = x^2 - p(x+1) - c$ , show that  $(\alpha+1)(\beta+1) = 1 - c$ .

**Ans.**

$$f(x) = x^2 - p(x+1) - c$$

$$= x^2 - px - (p+c)$$

$$\therefore \alpha + \beta = p \text{ and } \alpha\beta = -(p+c)$$

$$\text{Now } (\alpha+1)(\beta+1) = \alpha\beta + (\alpha + \beta) + 1$$

$$= -p - c + p + 1$$

$$= 1 - c$$

**7.** If the sum of the zeros of the quadratic polynomial  $f(t) = kt^2 + 2t + 3k$  is equal to their product, find the value of 'k'.

**Ans.**

$$f(t) = kt^2 + 2t + 3k$$

Sum of the zeros = Product of the zeros

$$\Rightarrow \frac{-2}{k} = \frac{3k}{k}$$

$$\Rightarrow k = -\frac{2}{3}$$

8. Divide  $(x^4 - 5x + 6)$  by  $(2 - x^2)$ .

Ans.

$$\begin{array}{r} -x^2 - 2 \\ 2 - x^2 \overline{) x^4 - 5x + 6} \\ \underline{x^4 + 2x^2} \phantom{+ 6} \\ 2x^2 - 5x + 6 \\ \underline{2x^2} \phantom{+ 6} \\ -5x + 10 \end{array}$$

Quotient =  $-x^2 - 2$

Remainder =  $-5x + 10$

9. Find the zeros of the polynomial  $p(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$  and verify the relationship between the zeros and its coefficients.

Ans.  $p(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

$$= 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$$

$$= 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$$

$$= (4x - \sqrt{3})(\sqrt{3}x + 2)$$

$\therefore$  zeros are  $4x - \sqrt{3} = 0$  and  $\sqrt{3}x + 2 = 0$

$$\Rightarrow x = \frac{\sqrt{3}}{4} \text{ and } x = -\frac{2}{\sqrt{3}}$$

$$\text{Sum of zeros} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= \left[ \frac{\sqrt{3}}{4} + \frac{(-2)}{\sqrt{3}} \right] = \frac{-5}{4\sqrt{3}}$$

$$\begin{aligned} \text{Product of zeros} &= \frac{\text{Constant term}}{\text{Coefficient of } x^2} \\ &= \frac{-2\sqrt{3}}{4\sqrt{3}} = \frac{-1}{2} \end{aligned}$$

**10. Find the value of 'k' so that the zeros of the quadratic polynomial  $3x^2 - kx + 14$  are in the ratio 7:6.**

**Ans.** Let the zeros are  $7p$  and  $6p$ .

$$3x^2 - kx + 14$$

$$\therefore 7p + 6p = \frac{-(-k)}{3} = \frac{k}{3}$$

$$\text{and } 7p \times 6p = \frac{14}{3}$$

$$\Rightarrow 42p^2 = \frac{14}{3}$$

$$p = 3$$

$$\Rightarrow 39p = k$$

$$\therefore k = 39 \times 3$$

$$\therefore k = 117$$

**11. If one zero of the quadratic polynomial  $f(x) = 4x^2 - 8kx - 9$  is negative of the other, find the value of 'k'.**

**Ans.**  $4x^2 - 8kx - 9$ , if one zero is  $\alpha$  then other is  $-\alpha$

$\therefore$  Sum of the zero = 0

$$\frac{8k}{4} = 0$$

$$\Rightarrow k = 0$$

12. Check whether the polynomial  $(t^2 - 3)$  is a factor of the polynomial  $2t^4 + 3t^3 - 2t^2 - 9t - 12$  by Division method.

Ans.

$$\begin{array}{r}
 2t^2 + 3t + 4 \\
 t^2 - 3 \overline{) 2t^4 + 3t^3 - 2t^2 - 9t - 12} \\
 \underline{- 2t^4} \qquad \qquad \qquad \underline{+ 6t^2} \\
 3t^3 + 4t^2 - 9t - 12 \\
 \underline{- 3t^3} \qquad \qquad \qquad \underline{+ 9t} \\
 4t^2 \qquad \qquad \qquad -12 \\
 \underline{- 4t^2} \qquad \qquad \qquad \underline{+ 12} \\
 0
 \end{array}$$

Yes,  $(t^2 - 3)$  is the factor of given polynomial.

CBSE Class 10 Mathematics

Important Questions

Chapter 2

Polynomials

3 Marks Questions

1. Apply division algorithm to find the quotient  $q(x)$  and remainder  $r(x)$  on dividing  $f(x)$  by  $g(x)$ , where  $f(x) = x^3 - 6x^2 + 11x - 6$ ,  $g(x) = x^2 + x + 1$

Ans.  $f(x) = g(x) \times q(x) + r(x)$

$$\begin{array}{r} x-7 \\ x^2+x+1 \overline{) x^3-6x^2+11x-6} \\ \underline{x^3+x^2+x} \phantom{-6} \\ -7x^2+10x-6 \\ \underline{+7x^2+7x+7} \\ -17x+1 \end{array}$$

$$\therefore (x^3 - 6x^2 + 11x - 6) = x^2 + 2x + 1(x - 7) + (17x + 1)$$

2. If two zeros of the polynomial  $x^4 - 6x^3 - 26x^2 + 138x - 35$  are  $2 \pm \sqrt{3}$ , find the other zeros.

Ans. Two zeros are  $2 \pm \sqrt{3}$

$\therefore$  Sum of zeros = 4

and product of the zeros = 1

$\therefore (x^2 - 4x + 1)$  is the factor of  $x^4 - 6x^3 - 26x^2 + 138x - 35$

$$\begin{array}{r}
 x^2 - 2x - 35 \\
 x^2 - 4x + 1 \sqrt{x^4 - 6x^3 - 26x^2 + 138x - 35} \\
 \underline{-x^4 + 4x^3 + x^2} \\
 -2x^3 - 27x^2 + 138x - 35 \\
 \underline{+ 2x^3 + 8x^2 + 2x} \\
 -35x^2 + 140x - 35 \\
 \underline{+ 35x^2 + 140x + 35} \\
 \hline
 0
 \end{array}$$

Now,

$$\begin{aligned}
 & x^2 - 2x - 35 \\
 &= x^2 - 7x + 5x - 35 \\
 &= x(x-7) + 5(x-7) \\
 &= (x-5)(x-7)
 \end{aligned}$$

∴ Zeros are

$$x = 7 \text{ and } x = -5$$

∴ Other two zeros are 7 and -5

3. What must be subtracted from the polynomial  $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$  so that the resulting polynomial is exactly divisible by  $g(x) = x^2 - 4x + 3$ ?

Ans.

$$\begin{array}{r}
 x^2 + 6x + 8 \\
 x^2 - 4x + 3 \sqrt{x^4 + 2x^3 - 13x^2 - 12x + 21} \\
 \underline{-x^4 + 4x^3 + 3x^2} \\
 6x^3 - 16x^2 - 12x + 21 \\
 \underline{-6x^3 + 14x^2 + 18x} \\
 8x^2 - 30x + 21 \\
 \underline{-8x^2 + 32x + 24} \\
 \hline
 2x - 3
 \end{array}$$

We must be subtract  $(2x - 3)$  to become a factor.

4. What must be added to  $6x^5 + 5x^4 + 11x^3 - 3x^2 + x + 5$  so that it may be exactly divisible by  $3x^2 - 2x + 4$ ?

Ans.

$$\begin{array}{r}
 x^2 + 6x + 8 \\
 3x^2 - 2x + 4 \overline{) 6x^5 + 5x^4 + 11x^3 - 3x^2 + x + 5} \\
 \underline{6x^5 - 4x^4 + 8x^3} \phantom{- 3x^2 + x + 5} \\
 9x^4 + 3x^3 - 3x^2 + x + 5 \\
 \underline{9x^4 - 6x^3 + 12x^2} \phantom{- 3x^2 + x + 5} \\
 9x^3 - 15x^2 + x + 5 \\
 \underline{9x^3 - 6x^2 + 12x} \phantom{- 3x^2 + x + 5} \\
 -9x^2 - 11x + 5 \\
 \underline{-9x^2 + 6x - 12} \phantom{- 3x^2 + x + 5} \\
 -17x + 17
 \end{array}$$

$$\begin{aligned}
 \text{So we must be added } & (3x^2 - 2x + 4) - (-17x + 17) \\
 & = 3x^2 - 2x + 4 + 17x - 17 \\
 & = 3x^2 + 15x - 13
 \end{aligned}$$

5. Find all the zeros of the polynomial  $f(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$ , if being given that two of its zeros are  $\sqrt{2}$  and  $-\sqrt{2}$ .

Ans.  $\sqrt{2}$  and  $-\sqrt{2}$  are the zeros.

$\therefore (x - \sqrt{2})(x + \sqrt{2})$  is the factor of the given polynomial.

$$\begin{array}{r}
 2x^2 - 3x + 1 \\
 x^2 - 2\sqrt{2x^4 - 3x^3 - 3x^2 + 6x - 2} \\
 \underline{-2x^4 \quad +4x^2} \\
 -3x^3 + x^2 + 6x - 2 \\
 \underline{+3x^3 \quad +6x} \\
 x^2 - 2 \\
 \underline{-x^2 \quad +2} \\
 0
 \end{array}$$

$$\begin{aligned}
 q(x) &= 2x^2 - 3x + 1 \\
 &= 2x^2 - 2x - x + 1 \\
 &= 2x(x-1) - 1(x-1) \\
 &= (2x-1)(x-1) \\
 \therefore \text{ other two zero's are}
 \end{aligned}$$

$$x = 1 \text{ and } x = \frac{1}{2}$$

6. On dividing  $x^3 - 3x^2 + x + 2$  by a polynomial  $g(x)$  the quotient and the remainder were  $(x - 2)$  and  $-2x + 4$  respectively, find  $g(x)$ .

$$\text{Ans. } p(x) = q(x) \times g(x) + r(x)$$

$$\begin{aligned}
 g(x) &= \frac{p(x) - r(x)}{q(x)} \\
 &= \frac{x^3 - 3x^2 + x + 2 + 2x - 4}{x - 2} \\
 &= \frac{x^3 - 3x^2 + 3x - 2}{x - 2}
 \end{aligned}$$

$$\begin{array}{r}
 x - 2\sqrt{x^3 - 3x^2 + 3x - 2} \\
 \underline{-x^3 \quad +2x^2} \\
 -x^2 + 3x - 2 \\
 \underline{+x^2 \quad -2x} \\
 x - 2 \\
 \underline{-x \quad +2} \\
 0
 \end{array}$$



$$g(x) = x^2 - x + 1$$

7. Find all zeros of  $f(x) = 2x^3 - 7x^2 + 3x + 6$  if its two zeros are  $-\sqrt{\frac{3}{2}}$  and  $\sqrt{\frac{3}{2}}$ .

Ans.  $f(x) = 2x^3 - 7x^2 + 3x + 6$

Two zeros are  $\pm\sqrt{\frac{3}{2}}$

$$\therefore \left(x + \sqrt{\frac{3}{2}}\right)\left(x - \sqrt{\frac{3}{2}}\right) = \frac{1}{2}(2x^2 - 3)$$

$\therefore (2x^2 - 3)$  is the factor of  $f(x)$ .

$$\begin{array}{r} x^2 - x - 2 \\ 2x^2 - 3 \overline{) 2x^3 - 7x^2 + 3x + 6} \\ \underline{-2x^4 \quad + 3x^2} \phantom{+ 6} \\ -2x^3 - 4x^2 + 3x + 6 \\ \underline{+2x^3 \quad + 3x} \phantom{+ 6} \\ -4x^2 + 6 \\ \underline{+4x^2 + 6} \\ 0 \end{array}$$

$$g(x) = x^2 - x - 2$$

$$= x^2 - 2x + x - 2$$

$$= x(x-2) + 1(x-2)$$

$$= (x+1)(x-2)$$

$\therefore$  other two zeros are

$$x+1=0 \text{ or } x=-1$$

and  $x-2=0$  or  $x=2$

$\therefore$  other two zeros are  $-1$  and  $2$

8. Obtain all zeros of the polynomial  $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$ , if two of its zeros are -2 and -1.

Ans.  $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$ , two zeros are -2 and -1

$\therefore (x+2)$  and  $(x+1)$  are the factors of  $f(x)$ .

$$\therefore (x+2)(x+1) = x^2 + 3x + 2$$

$$2x^2 - 5x - 3$$

$$\begin{array}{r}
 x^2 + 3x + 2 \sqrt{2x^4 + x^3 - 14x^2 - 19x - 6} \\
 \underline{2x^4 + 6x^3 + 4x^2} \\
 -5x^3 - 18x^2 - 19x - 6 \\
 \underline{+ 5x^3 + 15x^2 + 10x} \\
 -3x^2 - 9x - 6 \\
 \underline{+ 3x^2 + 9x + 6} \\
 0
 \end{array}$$

Now  $2x^2 - 5x - 3$

$$= 2x^2 - 6x + x - 3$$

$$= 2x(x-3) + 1(x-3)$$

$$= (x-3)(2x+1)$$

$\therefore$  zeros are

$$x-3=0$$

$$\Rightarrow x=3$$

and  $2x+1=0$

$$\Rightarrow x = -\frac{1}{2}$$

other two zeros are 3 and  $-\frac{1}{2}$

9. Obtain all other zeros of  $3x^4 + 6x^3 - 2x^2 - 10x - 5$ . If two of its zeros are  $\sqrt{\frac{5}{3}}$  and  $-\sqrt{\frac{5}{3}}$

Ans.  $3x^4 + 6x^3 - 2x^2 - 10x - 5$

zeros are  $\pm \sqrt{\frac{5}{3}}$

$\left(x + \sqrt{\frac{5}{3}}\right)\left(x - \sqrt{\frac{5}{3}}\right)$  is the factor given polynomial i.e.  $\left(x^2 - \frac{5}{3}\right) \frac{1}{3}(3x^2 - 5)$

$$\begin{array}{r}
 3x^2 - 5 \sqrt{3x^4 + 6x^3 - 2x^2 - 10x - 5} \\
 \underline{-3x^4 \quad + 5x^2} \\
 6x^3 + 3x^2 - 10x - 5 \\
 \underline{-6x^3 \quad + 10x} \\
 3x^2 - 5 \\
 \underline{-3x^2 \quad + 5} \\
 0
 \end{array}$$

Now  $x^2 + x + 1$

$$\Rightarrow (x+1)^2$$

$\therefore$  other zeros are

$$(x+1) = 0 \text{ and } x+1 = 0$$

$$x = -1 \text{ and } x = -1$$

$\therefore$  other two zeros -1 and -1

10. If the polynomial  $x^4 - 6x^3 + 16x^2 - 25x + 10$  is divided by another polynomial  $x^2 - 2x + k$ , the remainder comes out to be  $(x + a)$ , find 'k' and 'a'.

Ans.

$$\begin{array}{r}
 x^2 - 4x + (8 - k) \\
 x^2 - 2x + k \sqrt{x^4 - 6x^3 + 16x^2 - 25x + 10} \\
 \underline{-x^4 + 2x^3 - kx^2} \\
 -4x^3 + (16 - k)x^2 - 25x + 10 \\
 \underline{+4x^3 + 8x^2 - 4kx} \\
 (8 - k)x^2 + (4k - 25)x + 10 \\
 \underline{-(8 - k)x^2 + (16 - 2k)x + (8k - k^2)} \\
 (2k - 9)x + (k^2 - 8k + 10)
 \end{array}$$

but remainder is  $(x + a)$

$\therefore$  equating the coefficient of  $x$  and constant term.

$$\text{so } 2k - 8k + 10 = a$$

$$\Rightarrow 25 - 40 + 10 = a$$

$$\Rightarrow -5 = a$$

$$\therefore k = 5 \text{ and } a = -5$$

**11. Find the value of 'k' for which the polynomial  $x^4 + 10x^3 + 25x^2 + 15x + k$  is exactly divisible by  $(x + 7)$ .**

**Ans.**  $p(x) = x^4 + 10x^3 + 25x^2 + 15x + k$

$\therefore (x + 7)$  is the factor.

$$\therefore p(-7) = 0$$

$$\text{or } (-7)^4 + 10(-7)^3 + 25(-7)^2 + 15(-7) + k = 0$$

$$2401 - 3430 + 1225 - 105 + k = 0$$

$$k = 91$$

**12. If  $\alpha$  and  $\beta$  are the zeros of the polynomial  $f(x) = x^2 + px + q$ , find polynomial**

whose zeros are  $(\alpha + \beta)^2$  and  $(\alpha - \beta)^2$ .

Ans.  $f(x) = x^2 + px + q$ , if  $\alpha$  and  $\beta$  are zeros

$$\therefore \alpha + \beta = -p \text{ and } \alpha\beta = q$$

If zeros are  $(\alpha + \beta)^2$  and  $(\alpha - \beta)^2$

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

$$= (-p)^2 - 4q$$

$$(\alpha - \beta)^2 = -p^2 - 4q$$

Now sum of zeros

$$(\alpha + \beta)^2 + (\alpha - \beta)^2 = (-p)^2 + (p^2 - 4q)$$

$$= 2p^2 - 4q$$

Product of zeros

$$(\alpha + \beta)^2 (\alpha - \beta)^2 = (-p)^2 + (p^2 - 4q)$$

$$= 4p^4 - 4p^2q$$

$\therefore$  required polynomial is

$$x^2 - (\text{sum of zeros})x + \text{product of zeros}$$

$$= x^2 - (2p^2 - 4q)x + 4p^4 - 4p^2q$$

$$= x^2 - 2p^2x - 4qx + p^4 - 4p^2q$$

CBSE Class 10 Mathematics

Important Questions

Chapter 2

Polynomials

4 Marks Questions

1. Divide the polynomial  $p(x)$  by the polynomial  $g(x)$  and find the quotient and remainder in each of the following.

(i)  $p(x) = x^3 - 3x^2 + 5x - 3$ ,  $g(x) = x^2 - 2$

(ii)  $p(x) = x^4 - 3x^2 + 4x + 5$ ,  $g(x) = x^2 - x + 1$

(iii)  $p(x) = x^4 - 5x + 6$ ,  $g(x) = 2 - x^2$

Ans. (i)

$$\begin{array}{r} \phantom{x^2 - 2) } x - 3 \\ \hline x^2 - 2) x^3 - 3x^2 + 5x - 3 \\ \phantom{x^2 - 2) } \underline{\pm x^3} \phantom{+ 5x - 3} \\ \phantom{x^2 - 2) } -3x^2 + 7x - 3 \\ \phantom{x^2 - 2) } \phantom{-3x^2 + 7x - 3} \underline{\mp 3x^2} \phantom{+ 6} \\ \phantom{x^2 - 2) } \phantom{-3x^2 + 7x - 3} \phantom{\mp 3x^2 + 6} 7x - 9 \end{array}$$

Therefore, quotient =  $x - 3$  and Remainder =  $7x - 9$

(ii)

$$\begin{array}{r} \phantom{x^2 - x + 1) } x + x - 3 \\ \hline x^2 - x + 1) x^4 - 3x^2 + 4x + 5 \\ \phantom{x^2 - x + 1) } \underline{\pm x^4 \pm x^2} \phantom{+ 5} \mp x^3 \\ \phantom{x^2 - x + 1) } -4x^2 + 4x + 5 + x^3 \\ \phantom{x^2 - x + 1) } \phantom{-4x^2 + 4x + 5 + x^3} \underline{\mp x^2 \pm x} \phantom{+ 5} \pm x^3 \\ \phantom{x^2 - x + 1) } -3x^2 + 3x + 5 \\ \phantom{x^2 - x + 1) } \phantom{-3x^2 + 3x + 5} \underline{\mp 3x^2 \pm 3x \mp 3} \\ \phantom{x^2 - x + 1) } \phantom{-3x^2 + 3x + 5} \phantom{\mp 3x^2 \pm 3x \mp 3} 8 \end{array}$$



Therefore, quotient =  $x^2 + x - 3$  and, Remainder = 8

(iii)

$$\begin{array}{r}
 \phantom{-x^2+2)} \phantom{x^4-5x+6} \phantom{+} -x^2 - 2 \\
 \hline
 -x^2 + 2) \phantom{x^4} - 5x + 6 \\
 \phantom{-x^2+2)} \phantom{x^4} \phantom{-5x} + 2x^2 \\
 \hline
 \phantom{-x^2+2)} \phantom{x^4} - 5x + 6 + 2x^2 \\
 \phantom{-x^2+2)} \phantom{x^4} \phantom{-5x} + 4 + 2x^2 \\
 \hline
 \phantom{-x^2+2)} \phantom{x^4} - 5x + 10
 \end{array}$$

Therefore, quotient =  $-x^2 - 2$  and, Remainder =  $-5x + 10$

**2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial.**

(i)  $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii)  $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii)  $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Ans. (i)

$$\begin{array}{r}
 \phantom{x^2-3)} \phantom{2t^4+3t^3-2t^2-9t-12} \phantom{+} 2t^2 + 3t + 4 \\
 \hline
 x^2 - 3) 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 \phantom{x^2-3)} \phantom{2t^4} \phantom{+3t^3} \phantom{-2t^2} - 6t^2 \\
 \hline
 \phantom{x^2-3)} \phantom{2t^4} + 3t^3 + 4t^2 - 9t - 12 \\
 \phantom{x^2-3)} \phantom{2t^4} \phantom{+3t^3} \phantom{+4t^2} - 9t \\
 \hline
 \phantom{x^2-3)} \phantom{2t^4} + 4t^2 + 0 - 12 \\
 \phantom{x^2-3)} \phantom{2t^4} \phantom{+4t^2} \phantom{+0} - 12 \\
 \hline
 \phantom{x^2-3)} \phantom{2t^4} \phantom{+4t^2} \phantom{+0} 0
 \end{array}$$

∴ Remainder = 0

Hence first polynomial is a factor of second polynomial.

(ii)

$$\begin{array}{r} \phantom{x^2+3x+1) } 3x^2 - 4x + 2 \\ \hline x^2 + 3x + 1) 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\ \phantom{x^2+3x+1) } \underline{\pm 3x^4 \pm 9x^3 \pm 3x^2} \\ \phantom{x^2+3x+1) } -4x^3 - 10x^2 + 2x + 2 \\ \phantom{x^2+3x+1) } \phantom{-4x^3 - 10x^2 + 2x + 2} \underline{\mp 4x^3 \mp 12x^2 \mp 4x} \\ \phantom{x^2+3x+1) } \phantom{-4x^3 - 10x^2 + 2x + 2} \phantom{\mp 4x^3 \mp 12x^2 \mp 4x} + 2x^2 + 6x + 2 \\ \phantom{x^2+3x+1) } \phantom{-4x^3 - 10x^2 + 2x + 2} \phantom{\mp 4x^3 \mp 12x^2 \mp 4x} \phantom{+ 2x^2 + 6x + 2} \underline{\pm 2x^2 \pm 6x \pm 2} \\ \phantom{x^2+3x+1) } \phantom{-4x^3 - 10x^2 + 2x + 2} \phantom{\mp 4x^3 \mp 12x^2 \mp 4x} \phantom{+ 2x^2 + 6x + 2} \phantom{\pm 2x^2 \pm 6x \pm 2} 0 \end{array}$$

∴ Remainder = 0

Hence first polynomial is a factor of second polynomial.

(iii)

$$\begin{array}{r} \phantom{x^3-3x+1) } x^2 - 1 \\ \hline x^3 - 3x + 1) x^5 - 4x^3 + x^2 + 3x + 1 \\ \phantom{x^3-3x+1) } \underline{\pm x^5 \mp 3x^3 \mp x^2} \\ \phantom{x^3-3x+1) } -x^3 + 3x + 1 \\ \phantom{x^3-3x+1) } \phantom{-x^3 + 3x + 1} \underline{\mp x^3 + 3x \mp 1} \\ \phantom{x^3-3x+1) } \phantom{-x^3 + 3x + 1} \phantom{\mp x^3 + 3x \mp 1} 2 \end{array}$$

∴ Remainder ≠ 0

Hence first polynomial is not factor of second polynomial.

3. Obtain all other zeroes of  $(3x^4 + 6x^3 - 2x^2 - 10x - 5)$ , if two of its zeroes are  $\sqrt{\frac{5}{3}}$  and  $-\sqrt{\frac{5}{3}}$ .

Ans. Two zeroes of  $(3x^4 + 6x^3 - 2x^2 - 10x - 5)$  are  $\sqrt{\frac{5}{3}}$  and  $-\sqrt{\frac{5}{3}}$  which means that





$$p(x) = g(x) \cdot q(x) + r(x)$$

$$\Rightarrow x^3 - 3x^2 + x + 2 = g(x) \cdot (x-2) - 2x + 4$$

$$\Rightarrow x^3 - 3x^2 + x + 2 + 2x - 4 = g(x) \cdot (x-2)$$

$$\Rightarrow x^3 - 3x^2 + 3x - 2 = g(x) \cdot (x-2)$$

$$\Rightarrow g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$$

So, Dividing  $(x^3 - 3x^2 + 3x - 2)$  by  $(x - 2)$ , we get

$$\begin{array}{r} x^2 - x + 1 \\ \hline x - 2 \overline{) x^3 - 3x^2 + 3x - 2} \\ \underline{\pm x^3 \mp 2x^2} \phantom{- 2} \\ -x^2 + 3x - 2 \\ \underline{\mp x^2 \pm 2x} \phantom{- 2} \\ x - 2 \\ \underline{\pm x \mp 2} \\ 0 \end{array}$$

Therefore, we have  $g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2} = x^2 - x + 1$

**5. Give examples of polynomials  $p(x)$ ,  $g(x)$ ,  $q(x)$  and  $r(x)$ , which satisfy the division algorithm and**

**(i)  $\deg p(x) = \deg q(x)$**

**(ii)  $\deg q(x) = \deg r(x)$**

**(iii)  $\deg r(x) = 0$**

**Ans. (i)** Let  $p(x) = 3x^2 + 3x + 6$ ,  $g(x) = 3$



$$\begin{array}{r}
 x^2 + x + 2 \\
 \hline
 3) 3x^2 + 3x + 6 \\
 \pm 3x^2 \\
 \hline
 + 3x + 6 \\
 \pm 3x \\
 \hline
 + 6 \\
 \pm 6 \\
 \hline
 0
 \end{array}$$

So, we can see in this example that  $\deg p(x) = \deg q(x) = 2$

(ii) Let  $p(x) = x^3 + 5$  and  $g(x) = x^2 - 1$

$$\begin{array}{r}
 x \\
 \hline
 x^2 - 1) x^3 + 5 \\
 \pm x^3 \quad \mp x \\
 \hline
 x + 5
 \end{array}$$

We can see in this example that  $\deg q(x) = \deg r(x) = 1$

(iii) Let  $p(x) = x^2 + 5x - 3$ ,  $g(x) = x + 3$

$$\begin{array}{r}
 x + 2 \\
 \hline
 x + 3) x^2 + 5x - 3 \\
 \pm x^2 \pm 3x \\
 \hline
 + 2x - 3 \\
 \pm 2x \pm 6 \\
 \hline
 -9
 \end{array}$$

We can see in this example that  $\deg r(x) = 0$

**6. Find the zeroes of the following quadratic polynomials and verify the relationship**

between the zeros and the coefficients.

(i)  $x^2 - 2x - 8$

(ii)  $4s^2 - 4s + 1$

(iii)  $6x^2 - 3 - 7x$

(iv)  $4u^2 + 8u$

(v)  $t^2 - 15$

(vi)  $3x^2 - x - 4$

Ans. (i)  $x^2 - 2x - 8$

Comparing given polynomial with general form  $ax^2 + bx + c$ ,

We get  $a=1$ ,  $b=-2$  and  $c=-8$

We have,  $x^2 - 2x - 8$

$$= x^2 - 4x + 2x - 8$$

$$= x(x-4) + 2(x-4) = (x-4)(x+2)$$

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

$$(x-4)(x+2) = 0$$

$\Rightarrow x=4, -2$  are two zeroes.

$$\text{Sum of zeroes} = 4 - 2 = 2 = \frac{-(-2)}{1} = \frac{-b}{a} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = 4 \times -2 = -8 = \frac{-8}{1} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

(ii)  $4s^2 - 4s + 1$

Here,  $a=4$ ,  $b=-4$  and  $c=1$

We have,  $4s^2-4s+1$

$$=4s^2-2s-2s+1 = 2s(2s-1)-1(2s-1)$$

$$=(2s-1)(2s-1)$$

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

$$\Rightarrow (2s-1)(2s-1)=0$$

$$\Rightarrow s = \frac{1}{2}, \frac{1}{2}$$

Therefore, two zeroes of this polynomial are  $\frac{1}{2}, \frac{1}{2}$

$$\text{Sum of zeroes} = \frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-1)}{1} \times \frac{4}{4} = \frac{-(-4)}{4} = \frac{-b}{a} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of Zeroes} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

**(iii)**  $6x^2-3-7x$

Here,  $a=6$ ,  $b=-7$  and  $c=-3$

We have,  $6x^2-3-7x$

$$=6x^2-7x-3 = 6x^2-9x+2x-3$$

$$=3x(2x-3)+1(2x-3)=(2x-3)(3x+1)$$

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

$$\Rightarrow (2x-3)(3x+1)=0$$

$$\Rightarrow x = \frac{3}{2}, \frac{-1}{3}$$

Therefore, two zeroes of this polynomial are  $\frac{3}{2}, \frac{-1}{3}$

$$\text{Sum of zeroes} = \frac{3}{2} + \frac{-1}{3} = \frac{9-2}{6} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-b}{a} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of Zeroes} = \frac{3}{2} \times \frac{-1}{3} = \frac{-1}{2} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

**(iv)**  $4u^2+8u$

Here,  $a=4, b=8$  and  $c=0$

$$4u^2+8u = 4u(u+2)$$

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

$$\Rightarrow 4u(u+2)=0$$

$$\Rightarrow u=0, -2$$

Therefore, two zeroes of this polynomial are 0, -2

$$\text{Sum of zeroes} = 0-2=-2 = \frac{-2}{1} \times \frac{4}{4} = \frac{-8}{4} = \frac{-b}{a} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of Zeroes} = 0 \times -2 = 0 = \frac{0}{4} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

**(v)**  $t^2-15$

Here,  $a=1, b=0$  and  $c=-15$

We have,  $t^2-15$

$$\Rightarrow t^2=15$$

$$\Rightarrow t = \pm \sqrt{15}$$

Therefore, two zeroes of this polynomial are  $\sqrt{15}, -\sqrt{15}$

$$\text{Sum of zeroes} = \sqrt{15} + (-\sqrt{15}) = 0 = \frac{0}{1} = \frac{-b}{a} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of Zeroes} = \sqrt{15} \times (-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

**(vi)**  $3x^2 - x - 4$

Here,  $a=3, b=-1$  and  $c=-4$

We have,  $3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$

$$= x(3x-4) + 1(3x-4) = (3x-4)(x+1)$$

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

$$\Rightarrow (3x-4)(x+1) = 0$$

$$\Rightarrow x = \frac{4}{3}, -1$$

Therefore, two zeroes of this polynomial are  $\frac{4}{3}, -1$

$$\text{Sum of zeroes} = \frac{4}{3} + (-1) = \frac{4-3}{3} = \frac{1}{3} = \frac{-(-1)}{3}$$

$$= \frac{-b}{a} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of Zeroes} = \frac{4}{3} \times (-1) = \frac{-4}{3} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

**7. Find a quadratic polynomial each with the given numbers as the sum and product of**

its zeroes respectively.

(i)  $\frac{1}{4}, -1$

(ii)  $\sqrt{2}, 13$

(iii)  $0, \sqrt{5}$

(iv)  $1, 1$

(v)  $\frac{-1}{4}, \frac{1}{4}$

(vi)  $4, 1$

Ans. (i)  $\frac{1}{4}, -1$

Let quadratic polynomial be  $ax^2+bx+c$

Let  $\alpha$  and  $\beta$  are two zeroes of above quadratic polynomial.

$$\alpha+\beta = \frac{1}{4} = \frac{-b}{a}$$

$$\alpha \times \beta = -1 = \frac{-1}{1} \times \frac{4}{4} = \frac{-4}{4} = \frac{c}{a}$$

$$\therefore a = 4, b = -1, c = -4$$

$\therefore$  Quadratic polynomial which satisfies above conditions =  $4x^2-x-4$

(ii)  $\sqrt{2}, \frac{1}{3}$

Let quadratic polynomial be  $ax^2+bx+c$

Let  $\alpha$  and  $\beta$  be two zeros of above quadratic polynomial.



$$\alpha + \beta = \sqrt{2} \times \frac{3}{3} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$

$$\alpha \times \beta = \frac{1}{3} \text{ which is equal to } \frac{c}{a}$$

$$\therefore a = 3, b = -3\sqrt{2}, c = 1$$

$\therefore$  Quadratic polynomial which satisfies above conditions =  $3x^2 - 3\sqrt{2}x + 1$

**(iii)**  $0, \sqrt{5}$

Let quadratic polynomial be  $ax^2 + bx + c$

Let  $\alpha$  and  $\beta$  be two zeros of above quadratic polynomial.

$$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$$

$$\therefore a = 1, b = 0, c = \sqrt{5}$$

$\therefore$  Quadratic polynomial which satisfies above conditions =  $x^2 + \sqrt{5}$

**(iv)**  $1, 1$

Let quadratic polynomial be  $ax^2 + bx + c$

Let  $\alpha$  and  $\beta$  be two zeros of above quadratic polynomial.

$$\alpha + \beta = 1 = \frac{-(-1)}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$$

$$\therefore a = 1, b = -1, c = 1$$

$\therefore$  Quadratic polynomial which satisfies above conditions =  $x^2 - x + 1$

$$(v) \frac{-1}{4}, \frac{1}{4}$$

Let quadratic polynomial be  $ax^2 + bx + c$

Let  $\alpha$  and  $\beta$  be two zeros of above quadratic polynomial.

$$\alpha + \beta = \frac{-1}{4} = \frac{-b}{a}$$

$$\alpha \times \beta = \frac{1}{4} = \frac{c}{a}$$

$$\therefore a = 4, b = 1, c = 1$$

$\therefore$  Quadratic polynomial which satisfies above conditions =  $4x^2 + x + 1$

$$(vi) 4, 1$$

Let quadratic polynomial be  $ax^2 + bx + c$

Let  $\alpha$  and  $\beta$  be two zeros of above quadratic polynomial.

$$\alpha + \beta = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$$

$$\therefore a = 1, b = -4, c = 1$$

$\therefore$  Quadratic polynomial which satisfies above conditions =  $x^2 - 4x + 1$

**8. Verify that the numbers given alongside of the cubic polynomials below are their**



zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i)  $2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

(ii)  $x^3 - 4x^2 + 5x - 2; 2, 1, 1$

**Ans. (i)** Comparing the given polynomial with  $ax^3 + bx^2 + cx + d$ , we get

$a = 2, b = 1, c = -5$  and  $d = 2$ .

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$

$$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$$

$$= \frac{1+1-10+8}{0} = 0$$

$$p(1) = 2(1)^3 + (1)^2 - 5(1) + 2$$

$$= 2 + 1 - 5 + 2 = 0$$

$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$

$$= 2(-8) + 4 + 10 + 2$$

$$= -16 + 16 = 0$$

$\therefore \frac{1}{2}, 1$  and  $-2$  are the zeroes of  $2x^3 + x^2 - 5x + 2$ .

$$\text{Now, } \alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = \frac{1+2-4}{2} = \frac{-1}{2} = \frac{-b}{a}$$

And  $\alpha\beta + \beta\gamma + \gamma\alpha$

$$= \left(\frac{1}{2}\right)(1) + (1)(-2) + (-2)\left(\frac{1}{2}\right)$$

$$= \frac{1}{2} - 2 - 1 = \frac{-5}{2} = \frac{c}{a}$$

$$\text{And } \alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = -1 = \frac{-2}{2} = \frac{-d}{a}$$

(ii) Comparing the given polynomial with  $ax^3 + bx^2 + cx + d$ , we get

$$a = 1, b = -4, c = 5 \text{ and } d = -2.$$

$$p(2) = 2(2)^3 - 4(2)^2 + 5(2) - 2$$

$$= 8 - 16 + 10 - 2 = 0$$

$$p(1) = (1)^3 - 4(1)^2 + 5(1) - 2$$

$$= 1 - 4 + 5 - 2 = 0$$

$\therefore$  2, 1 and 1 are the zeroes of  $x^3 - 4x^2 + 5x - 2$ .

$$\text{Now, } \alpha + \beta + \gamma = 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$$

$$\text{And } \alpha\beta + \beta\gamma + \gamma\alpha = (2)(1) + (1)(1) + (1)(2)$$

$$= 2 + 1 + 2 = \frac{5}{1} = \frac{c}{a}$$

$$\text{And } \alpha\beta\gamma = 2 \times 1 \times 1 = 2 = \frac{-(-2)}{1} = \frac{-d}{a}$$

**9. Find a cubic polynomial with the sum of the product of its zeroes taken two at a time and the product of its zeroes are 2, -7, -14 respectively.**

**Ans.** Let the cubic polynomial be  $ax^3 + bx^2 + cx + d$  and its zeroes be  $\alpha, \beta$  and  $\gamma$ .

$$\text{Then } \alpha + \beta + \gamma = 2 = \frac{-(-2)}{1} = \frac{-b}{a} \text{ and } \alpha\beta + \beta\gamma + \gamma\alpha$$

$$= -7 = \frac{-7}{1} = \frac{c}{a}$$

$$\text{And } \alpha\beta\gamma = -14 = \frac{-14}{1} = \frac{d}{a}$$

Here,  $a = 1, b = -2, c = -7$  and  $d = 14$

Hence, cubic polynomial will be  $x^3 - 2x^2 - 7x + 14$ .

**10. If the zeroes of the polynomial  $x^3 - 3x^2 + x + 1$  are  $a - b, a, a + b$ , find  $a$  and  $b$ .**

**Ans.** Since  $(a - b), a, (a + b)$  are the zeroes of the polynomial  $x^3 - 3x^2 + 3x + 1$ .

$$\therefore \alpha + \beta + \gamma = a - b + b + a + b = \frac{-(-3)}{1} = 3$$

$$\Rightarrow 3a = 3$$

$$\Rightarrow a = 1$$

And  $\alpha\beta + \beta\gamma + \gamma\alpha$

$$= (a - b)a + a(a + b) + (a + b)(a - b) = \frac{1}{1} = 1$$

$$\Rightarrow a^2 - ab + a^2 + ab + a^2 - b^2 = 1$$

$$\Rightarrow 3a^2 - b^2 = 1$$

$$\Rightarrow 3(1)^2 - b^2 = 1 \quad [\because a = 1]$$

$$\Rightarrow 3 - b^2 = 1$$

$$\Rightarrow b = \pm 2$$

Hence  $a = 1$  and  $b = \pm 2$ .

**11. If the two zeroes of the polynomial  $x^4 - 6x^3 - 26x^2 + 138x - 35$  are  $2 \pm \sqrt{3}$ , find other zeroes.**

**Ans.** Since  $2 \pm \sqrt{3}$  are two zeroes of the polynomial  $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$ .

$$\text{Let } x = 2 \pm \sqrt{3}$$

$$\Rightarrow x - 2 = \pm \sqrt{3}$$

$$\text{Squaring both sides, } x^2 - 4x + 4 = 3$$

$$\Rightarrow x^2 - 4x + 1 = 0$$

Now we divide  $p(x)$  by  $x^2 - 4x + 1$  to obtain other zeroes.

$$\begin{array}{r} \phantom{x^2 - 4x + 1) } x^2 - 2x - 35 \\ \hline x^2 - 4x + 1) \ x^4 - 6x^3 - 26x^2 + 138x - 35 \\ \phantom{x^2 - 4x + 1) } \underline{\pm x^4 \mp 4x^3 \pm \phantom{x^2}} \\ \phantom{x^2 - 4x + 1) } \phantom{x^4} - 2x^3 - 27x^2 + 138x \\ \phantom{x^2 - 4x + 1) } \phantom{x^4} \underline{\mp 2x^3 \pm 8x^2 \mp 2x} \\ \phantom{x^2 - 4x + 1) } \phantom{x^4} \phantom{- 2x^3} - 35x^2 + 140x - 35 \\ \phantom{x^2 - 4x + 1) } \phantom{x^4} \phantom{- 2x^3} \underline{\mp 35x^2 \pm 140x \mp 35} \\ \phantom{x^2 - 4x + 1) } \phantom{x^4} \phantom{- 2x^3} \phantom{- 35x^2} 0 \end{array}$$

$$\therefore p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$$

$$= (x^2 - 4x + 1)(x^2 - 2x - 35)$$

$$= (x^2 - 4x + 1)(x^2 - 7x + 5x - 35)$$

$$= (x^2 - 4x + 1)[x(x - 7) + 5(x - 7)]$$

$$= (x^2 - 4x + 1)(x + 5)(x - 7)$$

$\Rightarrow (x + 5)$  and  $(x - 7)$  are the other factors of  $p(x)$ .

$\therefore -5$  and  $7$  are other zeroes of the given polynomial.

**12. If the polynomial  $x^4 - 6x^3 + 16x^2 - 25x + 10$  is divided by another polynomial  $x^2 - 2x + k$ , the remainder comes out to be  $x + a$ , find  $k$  and  $a$ .**

**Ans.** Let us divide  $x^4 - 6x^3 + 16x^2 - 25x + 10$  by  $x^2 - 2x + k$ .

$$\begin{array}{r}
 \phantom{x^2 - 2x + k} \overline{x^2 - 4x + (8 - k)} \\
 x^2 - 2x + k \overline{x^4 - 6x^3 + 16x^2 - 25x + 10} \\
 \phantom{x^2 - 2x + k} \underline{\pm x^4 \mp 2x^3 \pm kx^2} \\
 \phantom{x^2 - 2x + k} \phantom{x^4 - 6x^3 + 16x^2 - 25x + 10} -4x^3 + (16 - k)x^2 - 25x + 10 \\
 \phantom{x^2 - 2x + k} \phantom{x^4 - 6x^3 + 16x^2 - 25x + 10} \underline{\mp 4x^3 \pm 8x^2 \mp 4kx} \\
 \phantom{x^2 - 2x + k} \phantom{x^4 - 6x^3 + 16x^2 - 25x + 10} \phantom{-4x^3 + (16 - k)x^2 - 25x + 10} (8 - k)x^2 + (5k - 25)x + 10 \\
 \phantom{x^2 - 2x + k} \phantom{x^4 - 6x^3 + 16x^2 - 25x + 10} \phantom{-4x^3 + (16 - k)x^2 - 25x + 10} \phantom{-4x^3 + (16 - k)x^2 - 25x + 10} \underline{\mp (8 - k)x^2 \mp 2(8 - k)x \pm (8 - k)k} \\
 \phantom{x^2 - 2x + k} \phantom{x^4 - 6x^3 + 16x^2 - 25x + 10} \phantom{-4x^3 + (16 - k)x^2 - 25x + 10} \phantom{-4x^3 + (16 - k)x^2 - 25x + 10} \phantom{-4x^3 + (16 - k)x^2 - 25x + 10} (2k - 9)x - (8 - k)k + 10
 \end{array}$$

$$\therefore \text{Remainder} = (2k - 9)x - (8 - k)k + 10$$

On comparing this remainder with given remainder, i.e.  $x + a$ ,

$$2k - 9 = 1$$

$$\Rightarrow 2k = 10$$

$$\Rightarrow k = 5$$

$$\text{And } -(8 - k)k + 10 = a$$

$$\Rightarrow a = -(8 - 5)5 + 10 = -5$$